FEEDBACK REDUCTION BY THRESHOLDING IN MULTI-USER BROADCAST CHANNELS: DESIGN AND LIMITS

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ABSTRACT

To utilize the multi-user diversity in broadcast channels, the channel state information (CSI) of each user must be known at the transmitter. To reduce the overhead of CSI feedback under random beamforming the question of which receivers should feed back their CSI is investigated. Using the closed form expression for the SINR distribution, thresholding functions $T(n)$ are designed to meet specific design criterion as a function of the number of receivers. Specifically three design criterion are proposed. The asymptotic limits of the successful thresholding functions $T(n)$ are found. If $T(n)$ scales slower than $\log n$, asymptotically no performance is lost. If $T(n)$ scales faster than $\log n$, all multi-user diversity is lost.

Index Terms:: multi-user MIMO, broadcast channel, random beamforming, limited feedback, thresholding

1. INTRODUCTION

The multi-user broadcast channel has been the subject of intense study for many years as it has numerous direct applications to modern technologies such as the cell phone downlink channel. Great strides have been taken in characterizing the theoretical limits of the broadcast channel, especially the vector Gaussian broadcast channel. Specifically there has been an explosion of work regarding the sum-rate capacity of the vector Gaussian broadcast channel starting with [1]. One of the greatest hindrances in achieving the theoretical optimum is having complete CSI for all of the users in the multi-user broadcast channel available at the transmitter. It is well known that without CSI at the transmitter, the system cannot utilize multi-user diversity, which is a significant benefit of multi-user systems. In order to reap the benefits of the multi-user architecture, CSI feedback from the receivers to the transmitter is essential.

The first steps towards thresholding feedback are in the works [2, 3, 4] which consider one bit feedback schemes where a binary decision is made as to whether or not the channel is in a good or bad state for transmission. These works were later extended in [5], where the feedback channel is used multiple times per coherence time to allow for refinement of the scheduling decision. The rate enhancement due to additional feedback bits is analyzed in [6]. Additional work along these lines can be found in [7] as well as an overview of feedback systems in general in [8].

This contribution addresses the question of which user should feed back CSI under the random beamforming transmit scheme proposed in [9] and extended in [10] in order to reduce CSI feedback. Specifically, the goal of this work is to design thresholds such that a user feeds back its observed SINR value if the value is above the threshold and does not feed back if the SINR is below the threshold. The design of the thresholds is addressed in two parts. The first part considers design of optimal thresholds as a function of the number of users in the system under under three proposed performance metrics. The second part analyzes the scaling rate of any successful threshold asymptotically in the number of users.

The organization of the contribution is as follows: the system model and random beamforming transmit scheme are discussed in Section 2. Section 3 discusses the effects of thresholding on the SINR distribution. Section 4 addresses the design of the thresholding function with respect to certain performance metrics. Section 5 analyzes the asymptotic scaling rate of the thresholding function. Finally, Section 6 summarizes the results of this contribution.

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2. SYSTEM MODEL AND RANDOM BEAMFORMING

In this section the system model is described. Under the given system model, the random beamforming technique first proposed in [9] is outlined. The key implications of this work will be discussed.

2.1. System Model

A block fading channel model is assumed for the Gaussian broadcast channel. The transmitter has M transmit antennas and there are n receivers (users), each with a single receive antennas. It is assumed that $n \gg M$ which is typically the case in a cellular system. Let $\mathbf{s}(t) \in \mathbb{C}^{M\times 1}$ be the transmitted vector of symbols at time slot t and $y_i(t) \in \mathbb{C}$ be the received symbol at the i^{th} user at time slot t. Under the block fading assumption, the time slot t notation will be suppressed with the understanding the channel is constant and known during each scheduling epoch. The following model is used for the input-output relationship between the transmitter and the i^{th} user:

$$\mathbf{y}_i = \sqrt{\rho_i}\mathbf{h}_i^H\mathbf{s} + w_i, \quad i = 1, \dots, n. \tag{1}$$

$\mathbf{h}_i \in \mathbb{C}^M$ is the complex channel vector which is assumed to be known at the receiver, $w_i \in \mathbb{C}$ is the complex white additive noise, and the elements of $\mathbf{h}$ and w_i are i.i.d. complex Gaussians with zero mean and unit variance and the superscript H denotes the conjugate transpose. This is the standard Rayleigh fading model, where the channel gain between every transmit-receive antenna pair is a complex circularly symmetric normal random variable. The transmit power is chosen to be M, i.e. $E\{\mathbf{s}^*\mathbf{s}\} = M$, so that the normalized power per antenna is one. The signal-to-noise ratio (SNR) at the receiver is $E\{\rho_i|\mathbf{h}_i^H\mathbf{s}|^2\} = M\rho_i$ and ρ_i is the SNR of the i^{th} user. It is assumed that the network is homogeneous, thus $\rho_i = \rho\, \forall i$.

2.2. Random Beamforming

The random beamforming transmit scheme involves generating an M dimensional random orthonormal basis from an isotropic distribution, with basis vectors $\boldsymbol{\phi}_m \in \mathbb{C}^{M\times 1}$ for $m = 1, \dots, M$. Let s_m

be the m^{th} transmit symbol, then the total transmit signal is given by

$$\mathbf{s} = \sum_{m=1}^{M} \phi_m s_m. \tag{2}$$

The received signal at the i^{th} user is given by

$$y_i = \sum_{m=1}^{M} \sqrt{\rho}\mathbf{h}_i^H \phi_m s_m + w_i. \tag{3}$$

Every receiver measures and feeds back the SINR for each of the M transmit directions.The transmitter then schedules the users experiencing the largest SINR for each transmit direction.

It is assumed that the i^{th} user knows the quantity $\mathbf{h}_i^H \phi_m$ from Equation 3 for each transmit direction. With this knowledge, the SINR observed at the i^{th} user for the m^{th} transmit direction is given by the following equation:

$$SINR_{i,m} = \frac{|\mathbf{h}_i^H \phi_m(t)|^2}{\frac{2}{\rho} + \sum_{k \neq m} |\mathbf{h}_i^H \phi_k(t)|^2}. \tag{4}$$

The orthonormal beamforming vectors and i.i.d. zero mean and unit variance complex Gaussian elements of $\mathbf{h}_i$ imply the numerator in Equation 4 is distributed as a $\chi^2(2)$ random variable and the denominator is an independent $\chi^2(2(M-1))$ random variable. It is shown in [9] that the distribution of the SINRs at the receive antennas is given by

$$f_{SINR}(x) = \frac{e^{-\frac{x}{\rho}}}{(1+x)^M}\left(\frac{1}{\rho}(1+x) + M - 1\right)u(x), \tag{5}$$

where $u(x)$ is the unit step function. From now on, the $u(x)$ will be dropped from the SINR distribution and SINR density expressions with the understanding that all the SINR random variables of interest are nonnegative. Integrating the density in Equation 5 yields the distribution function for the SINR random variables:

$$F_{SINR}(x) = 1 - \frac{e^{-\frac{x}{\rho}}}{(1+x)^{M-1}}. \tag{6}$$

The main result of [9] is that asymptotically random beamforming has the optimal sum-rate throughput scaling. More specifically, the sum-rate throughput of the random beamforming technique R asymptotically scales at a rate of $M \log\log n$, which is the theoretical optimal scaling rate. This theorem is now stated:

Theorem 1. *([9]) Let M and ρ be fixed and $N = 1$. Then*

$$\lim_{n\to\infty} \frac{R}{M \log\log n} = 1 \tag{7}$$

where R is the throughput of the random beamforming technique.

With the previous results now reviewed, the properties of the distribution function can be used to ask questions about designing thresholds to limit which users feed back CSI and the effects these thresholds have on the system performance.

3. SINR DISTRIBUTION WITH THRESHOLDS

The effects of thresholding on the distribution function F_{SINR} given by Equation 6 must be understood in order to design a thresholds to meet certain performance conditions. In [10], it is observed that that

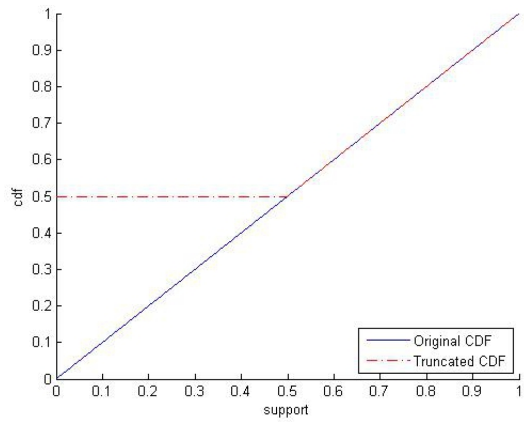

Fig. 1. Uniform Distribution Truncated at $x_{threshold} = 0.5$

the threshold truncates the distribution function of the SINR values fed back to the transmitter in the following manner:

$$F_{SINR}^{threshold}(x) = \begin{cases} F_{SINR}(x) & x \geq x_{threshold} \\ F_{SINR}(x_{threshold}) & 0 \leq x < x_{threshold} \\ 0 & x < 0 \end{cases} \tag{8}$$

Equation 8 states that the probability mass less than the threshold value $x_{threshold}$ concentrates at zero, and this is the probability that the user does not feed back. The mass concentrates at zero because this is the lowest possible value that the nonnegative SINR random variables can attain. With this mapping, the values that do not exceed the threshold do not affect the greedy scheduling decision since they are assumed to be the smallest possible value. To generalize the truncation from nonnegative random variables, let $x_1 = \inf\{x : F(x) > 0\}$ for an arbitrary distribution function F, i.e. the left end point of the support of F. Thresholding an arbitrary distribution function produces the following truncation:

$$F^{threshold}(x) = \begin{cases} F(x) & x \geq x_{threshold} \\ F(x_{threshold}) & x_1 \leq x < x_{threshold} \\ 0 & x < x_1 \end{cases} \tag{9}$$

Figure 1 shows the truncation of the uniform distribution at $x_{threshold} = 0.5$. The probability mass less than $x_{threshold} = 0.5$ maps to the left end point of the support, which in this case is $x_1 = 0$.

Let X be a random variable drawn from a distribution function F. Then the distribution function from Equation 9 corresponds to the following random variable Y:

$$Y = \begin{cases} X & X \geq x_{threshold} \\ x_1 & X < x_{threshold} \end{cases} \tag{10}$$

From an engineering perspective, this formulation is motivated by the following scenario. Let there be n users, where user i makes an observation X_i drawn i.i.d. from F. Each user decides to send back their observation X_i to the scheduler if it exceeds some threshold $x_{threshold}$. The scheduler has the task of finding the maximum observation. The users that did not report their observations because they did not exceed the threshold should not effect the decision of finding the maximum. Thus if a user did not report a value, their observation is assumed to be x_1. By taking on the lowest possible value, it cannot affect the task of finding the maximum value among the users that do feed back, and this motivates Equation 10.

In [10], it is shown that any finite threshold does not affect the asymptotic performance. The goal of this contribution is to reduce CSI feedback by designing a threshold as a function of the number of users, $T(n)$, that meet specified performance goals.

4. THRESHOLDS FOR A FINITE NUMBER OF USERS

The design of an appropriate threshold depends on the performance metrics that are to be optimized. This section discusses how to design the threshold for a system with n users to address the following metrics:

1. What should the threshold be so that the probability that no user feeds back information for any transmit beam is less than some design parameter γ_{outage}?
2. What should the threshold be so that on average only k users feed back SINR information for every transmit beam?
3. What should the threshold be so that the rate loss compared to the unthresholded system is less than some design parameter R_{loss}?

The first question is motivated by the fact that for any threshold, as long as one users feeds back SINR information for each transmit beam, then the maximum SINR is conveyed at the transmitter. In the event that the threshold is chosen so that no user feeds back SINR information for a transmit beam, multi-user diversity is lost on that beam, thus the probability that no user feeds back information needs to be controlled. The design parameter γ_{outage} is the "multi-user diversity outage probability", which is the maximum proportion of the time that the system is allowed to have no user feedback CSI information for any transmit direction. The second question considers the design of the threshold to constrain the average number of users feeding back. The final question considers designing the threshold to trade off reducing sum-rate throughput to reduce the amount of CSI feedback. The following subsections provide solutions to the threshold design questions just raised.

4.1. Designing a Threshold under Outage Constraints

The first design criterion corresponds to each transmit beam having fed back SINR information $1-\gamma_{outage}$ percent of the time. The system is in multi-user diversity outage if at least one transmit beam has no feedback information, and this situation should occur less than γ_{outage} percent of the time. The key to the analysis of designing a threshold to meet this requirement is to recall from Equation 8 the effect thresholding has on the underlying distribution function.

Since the SINR is always positive, if $0 \leq SINR < x_{threshold}$, then the user does not feed back the SINR information for that particular transmit beam, and this happens with probability $F_{SINR}(x_{threshold})$. For a system with n users and M transmit beams, let $Y_{i,j}, i \in \{1,\ldots,n\}, j \in \{1,\ldots,M\}$ be a Bernoulli random variable indicating that user i feeds back SINR information for transmit beam j. From the previous statement, $Y_{i,j}$ are i.i.d. with the following distribution:

$$\begin{aligned}\Pr\left[Y_{i,j}=1\right] &= 1-\Pr\left[Y_{i,j}=0\right] \\ &= 1-F_{SINR}\left(x_{threshold}\right). \end{aligned} \qquad (11)$$

Define the random variable $Z_j = Y_{1,j}+\cdots+Y_{n,j}$, which denotes the total number of users that feedback information for transmit beam j. Since the $Y_{i,j}$ are i.i.d. for all i, Z_j is a binomial

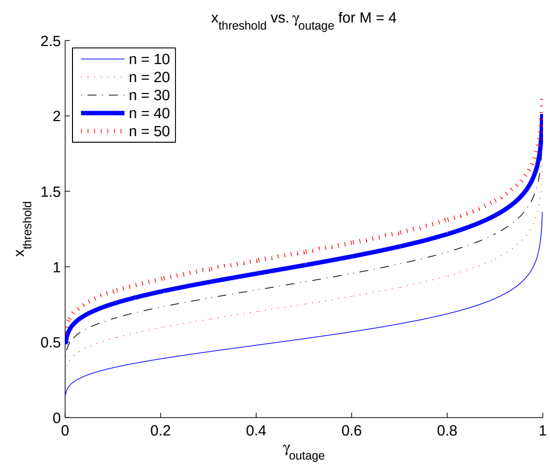

Fig. 2. Threshold as a function of γ_{outage} for $M=4$ and $\rho=1$

random variable, i.e. $Z_j \sim B\left(n, 1-F_{SINR}\left(x_{threshold}\right)\right)$. Outage occurs for transmit beam j when $Z_j=0$, and $\Pr\left[Z_j=0\right] = \left(1-\left(1-F_{SINR}(x_{threshold})\right)\right)^n = \left(F_{SINR}(x_{threshold})\right)^n$. The probability that the system is in outage is thus given by:

$$\begin{aligned}\Pr[\text{outage}] &= \gamma_{outage} = 1-\Pr[\text{not in outage}] \\ &= 1-\Pr\left[Z_1\neq 0,\ldots,Z_N\neq 0\right] \\ &= 1-\prod_{i=1}^{M}\Pr\left[Z_i\neq 0\right] = 1-\prod_{i=1}^{M}\left(1-\Pr\left[Z_i=0\right]\right) \\ &= 1-\left(1-\left(F_{SINR}(x_{threshold})\right)^n\right)^M. \end{aligned}$$

Therefore for a fixed number of users n, a fixed number of transmit antenna M, and design parameter $\gamma_{outage}\in[0,1]$, the threshold can be determined:

$$\begin{aligned}&\gamma_{outage} = 1-\left(1-\left(F_{SINR}(x_{threshold})\right)^n\right)^M \Rightarrow \\ &T(n) = x_{threshold} = F_{SINR}^{-1}\left(\left(1-\left(1-\gamma_{outage}\right)^{\frac{1}{M}}\right)^{\frac{1}{n}}\right) \end{aligned} \qquad (12)$$

where F_{SINR}^{-1} is the inverse distribution function. The solution to the inverse distribution function can be expressed in terms of Lambert functions. An alternative solution to evaluating the inverse is to use an iterative algorithm. Because the distribution function is a function of a scalar and monotonic, iterative algorithms are well suited for the computation of the inverse. Figure 2 shows the value of $x_{threshold}$ as a function of γ_{outage} for varying numbers of users in the system. As expected, for γ_{outage} approaching zero, the threshold also has to go to zero, and for γ_{outage} approaching one, the threshold begins to blow up. For larger numbers of users in the system, the threshold can be set higher and achieve the same outage probability due to the increased maximum order statistics.

In the unthresholded system, each receiver feeds back M SINR values for a total of nM values in the system. With the threshold given by Equation 12 and the probability given by Equation 11, on average only $nM\left(1-F_{SINR}(x_{threshold})\right)$ SINR values are fed back, or an average reduction of $nMF_{SINR}(x_{threshold})$ SINR values.

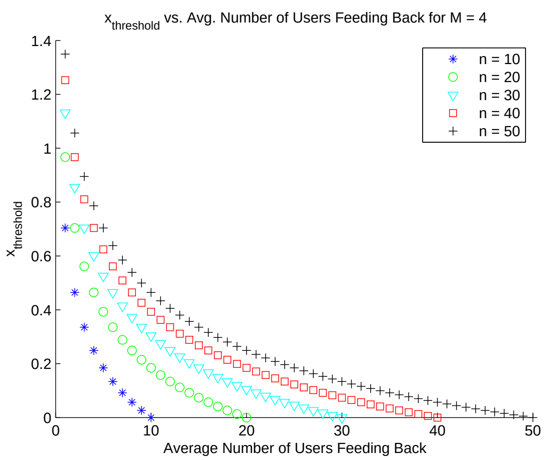

Fig. 3. Threshold as a function of the Average Number of Users Feeding Back for $M = 4$ and $\rho = 1$

4.2. Designing a Threshold Constraining the Average Number of Users Feeding Back

Because the SINR information is distributed across geographically separated receivers, it is impossible to design a threshold that can guarantee that exactly $k < n$ users feed back information for each transmit beam without further sharing of information. The best one can hope to do is use the statistical information about the channel metric, the observed SINR, to create a threshold that guarantees on average only k users feed back. From Section 4.1 it is known that the probability of feeding back is a Bernoulli random variable, and the number of users feeding back for a particular transmit beam is a binomial random variable with parameters n and $1 - F_{SINR}(x_{threshold})$. With these observations, the design of the threshold under the average number of users feeding back per transmit beam metric is straightforward. Once again, let $Z_j \sim B(n, 1 - F_{SINR}(x_{threshold}))$ be the number of users feeding back for transmit beam j. Then

$$\begin{aligned} k &= E\left[\text{\# of users feeding back for transmit beam} j\right] \\ &= E[Z_j] = n\left(1 - F_{SINR}\left(x_{threshold}\right)\right) \\ &\Rightarrow T(n) = x_{threshold} = F_{SINR}^{-1}\left(\frac{n-k}{n}\right). \end{aligned} \quad (13)$$

Figure 3 shows the threshold as a function of the constraint on the average number of users feeding back for varying number of users in the system. As the average number of users allowed is increased, the threshold decreases as expected to accommodate more users exceeding the threshold. When the average number of users equals the number of users in the system, the threshold is equal to zero.

4.3. Designing a Threshold with Rate Loss Constraints

A third design constraint of interest is to select a threshold that bounds the rate lost in the system as compared to the system without a threshold. Let R be the throughput of the unthresholded system. From [9, 10], it is known for the random beamforming scheme that R can be expressed as

$$R \approx M \int_0^\infty \frac{1}{1+x}\left(1 - \left(F_{SINR}(x)\right)^n\right) dx, \quad (14)$$

where the approximation symbol becomes very accurate for a moderate number of users in the system. Equation 14 is equivalent to writing out the integral equation to evaluate

$$ME\left[\log\left(1 + \max\left\{X_1, \ldots, X_n\right\}\right)\right], \quad (15)$$

where the X_i are drawn i.i.d. from F_{SINR}. The exponent of n on the distribution function is the effect of the maximum order statistics.

The sum-rate throughput of the thresholded system is more complicated and consists of two parts: the rate when a transmit direction receives CSI and the rate when no user feeds back information for a transmit direction and thus a random user is scheduled on that beamforming vector. Analogous to Equation 14, the sum-rate throughput of the thresholded system can be shown to be given by

$$R_{threshold} \approx M \int_0^\infty \frac{1}{1+x}\left(1 - F_{threshold}(x)\right) dx, \quad (16)$$

where

$$F_{threshold} = \begin{cases} F_{SINR}(x)^n \text{ for } x \in [x_{threshold}, \infty) \\ F_{SINR}(x_{threshold})^n \cdot F_{random}(x) \\ \text{for } x \in [0, x_{threshold}) \end{cases} \quad (17)$$

and

$$F_{random}(x) = \frac{F_{SINR}(x)}{F_{SINR}(x_{threshold})} \text{ for } x \in [0, x_{threshold}). \quad (18)$$

Equation 17 reflects the two scenarios just described.

Let $R_{loss} = R - R_{threshold} \in [0, R_{random}]$ be the design parameter that quantifies the amount of rate that can be sacrificed in order to reduce feedback. The upper bound on R_{loss} is R_{random} since in the worse case scenario this throughput can be achieved by randomly scheduling users. Since F_{SINR} for this system is continuous and monotonically increasing, R_{loss} is monotonic increasing as a function of $x_{threshold}$ and there exists one solution for $x_{threshold}$ that solves the above equation for R_{loss}. R_{loss} is a complicated function of $x_{threshold}$, but due to its monotonicity, an iterative algorithm can be used to find $x_{threshold}$ to sufficient accuracy. Figure 4 shows the threshold as a function of the design parameter R_{loss}. For small values of R_{loss}, the larger the number of users, the higher the threshold can be set and still achieve the same rate loss, which is expected since for a larger number of users, the probability of exceeding the threshold increases. All the thresholds eventually approach infinity as R_{loss} approaches the unthresholded sum-rate throughput given by Equation 14.

5. THRESHOLDING ASYMPTOTICALLY IN THE NUMBER OF USERS

In the previous section, the optimal thresholding function $T(n)$ was determined for three performance metrics. In this section, the question analyzed is how fast can the thresholding function $T(n)$ grow as a function of the number of users and still have the thresholded system achieve the same optimal asymptotic sum-rate scaling as the non-thresholded system? The results on the scaling rate of successful thresholding functions $T(n)$ will now be summarized.

To briefly review some notation, a function f is said to be dominated by g asymptotically if as $n \to \infty$, eventually $|f(n)| \leq$

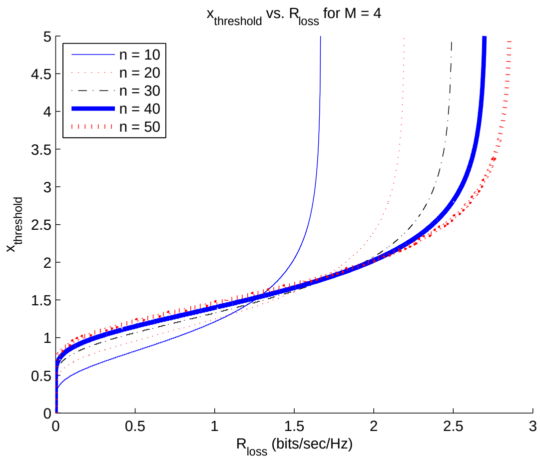

Fig. 4. Threshold as a function of R_{loss} for $M = 4$ and $\rho = 1$

$|g(n)| \cdot \epsilon$ for every $\epsilon > 0$ and is denoted $f(n) \in o(g(n))$. Also, f is said to asymptotically dominate g if as $n \to \infty$, eventually $|f(n)| \geq |g(n)| \cdot \epsilon$ for every $\epsilon > 0$ and is denoted $f(n) \in \omega(g(n))$. Having established this notation, the main result on the sufficient condition of the scaling rate of $T(n)$ is now stated.

Theorem 2. ***Sufficient Scaling Condition*** *Under the Rayleigh fading model and random beamforming scheme described in Section 2, if the thresholding function $T(n) \in o(\log n)$, then the sum-rate throughput of the system satisfies $\frac{R}{M \log \log n} \to 1$.*

Theorem 2 says that as long as $T(n)$ grows slower than $\log n$, that asymptotically the sum-rate throughput of the thresholded system still achieves the optimal sum-rate throughput scaling rate of $M \log \log n$. The next question to ask is can the thresholding function $T(n)$ grow too fast such that asymptotically no user exceeds the threshold. This question is answered affirmatively in the following theorem:

Theorem 3. ***Necessary Scaling Condition*** *Let the SINR observed at each user be distributed i.i.d. according to F_{SINR}. Then if $T(n) \in \omega(\log n)$, asymptotically no user exceeds the threshold and consequently all multi-user diversity is lost.*

The results of Theorem 2 and 3 provide the limits of the asymptotic scaling rate of any successful thresholding function $T(n)$. The form of Theorem 1 and Equation 15 can portend these results. Theorem 1 says the sum-rate scaling rate of random beamforming is $M \log \log n$ while the sum-rate throughput of random beamforming is given by Equation 15, and thus one would expect asymptotically $\mathbb{E}[\max\{X_1, \dots X_n\}]$ to roughly grow as $\log n$. Informally, this is why when $T(n) \in o(\log n)$ the asymptotic sum-rate scaling rate is preserved, and when $T(n) \in \omega(\log n)$ eventually the threshold is so large no user exceeds it.

6. CONCLUSION

This paper addresses the question of which user should feed back the observed SINR given statistical knowledge of the channel metric. When every receiver feeds back their SINR information, it is shown in [9] that random beamforming achieves the optimal sum-rate throughput scaling. In [10], it is shown that any finite threshold such that users experiencing SINRs below this threshold do not feed back does not affect this optimal scaling rate. The questions raised in this contribution discuss how the thresholds can be designed as a function of the number of users in the system to achieve certain performance metrics.

Using the statistics of the channel metric, thresholds for an n user system under three different design criterion are found. Under the first criterion, a threshold is designed to limit the probability that no user in the system feeds back CSI information for any transmit beam to be less than the design parameter γ_{outage}. The second criterion considers choosing a threshold such that on average only k out of n users in the system feed back CSI information for each transmit beam. Finally, a threshold is designed to limit the rate lost compared to a random scheduling algorithm to be below a design parameter R_{loss}.

Lastly, the question of how fast can the threshold grow yet still exhibit the optimal scaling is considered. It is shown that any thresholding function $T(n)$ that grows slower than $\log n$ as a function of the number of users n, i.e. $T(n) \in o(\log n)$, achieves the optimal scaling rate. Conversely, any thresholding function $T(n) \in \omega(\log n)$ causes the system to lose all multi-user diversity.

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